Sharpening Low-Energy, Standard-Model Tests via Correlation Coefficients in Neutron β-Decay

S. Gardner* and C. Zhang

Department of Physics and Astronomy,

University of Kentucky, Lexington, KY 40506-0055

(March 21, 2001)

The correlation coefficients a, A, and B in neutron β-decay are proportional to the ratio of the axial-vector to vector weak coupling constants, g_A/g_V, to leading recoil order. With the advent of the next generation of neutron decay experiments, the recoil-order corrections to these expressions become experimentally accessible, admitting a plurality of Standard Model (SM) tests. The measurement of both a and A, e.g., allows one to test the conserved-vector-current (CVC) hypothesis and to search for second-class currents (SCC) independently. The anticipated precision of these measurements suggests that the bounds on CVC violation and SCC from studies of nuclear β-decay can be qualitatively bettered. Departures from SM expectations can be interpreted as evidence for non-$V-A$ currents.

Precision nuclear β-decay measurements have played an important role in the rise of the Standard Model (SM), giving strong credence to the conserved-vector-current (CVC) hypothesis, as well as to the absence of second-class currents (SCC). We show that upcoming neutron-decay experiments can sharpen tests of the CVC hypothesis and of the absence of SCC significantly, eliminating assumptions inherent to the nuclear studies.

Searches for CVC violation and SCC in nuclear β-decay experiments have spanned decades of effort. We consider a CVC test originally suggested by Gell-Mann [1]: the strength of the "weak magnetism" term of the nucleon weak current ought be given by the strength of the corresponding electromagnetic M1 transition. The SM test realized from such a comparison constrains a combination of the weak magnetism and induced tensor terms of the nucleon weak current. The induced tensor term is a "second class" current and thus is zero in the SM [2], save for isospin-violating effects engendered by the differing mass and charge of the u and d quarks. In tests of this sort, the CVC hypothesis is tested if SCC are assumed to be zero, or, alternatively, the non-existence of SCC is tested if the CVC hypothesis is assumed to be valid.

Historically, the best constraints on the non-existence of SCC and CVC violation are realized in the mass 12 system [3,4]. The CVC hypothesis can be tested through the comparison of the spectral shape correction parameters $a_\mp$ measured in $^{12}\mathrm{B} \to ^{12}\mathrm{C}$ and $^{12}\mathrm{N} \to ^{12}\mathrm{C}$ transitions with the strength of the electromagnetic M1 transition from the analog state of $^{12}\mathrm{C}$. This procedure yields a test of the CVC hypothesis at the 10% level [3–5]. In order to realize a SCC test, the decays of spin-aligned $^{12}\mathrm{B}$ and $^{12}\mathrm{N}$ nuclei are studied. For purely aligned $1^+ \to 0^+$ transitions [6], the $e^\mp$ angular distribution for $^{12}\mathrm{B}$ $(-)$ and $^{12}\mathrm{N}$ $(+)$ decay is given by [4]

$$W_\mp(E_e,\theta,\mathcal{A}) \propto p_e E_e (E_e - E_e^{\max})^2 [1 + \mathcal{A}\alpha_\mp P_2(\cos\theta)],$$

where p_e and E_e are the momentum and energy of the electron (positron), $E_e^{\max}$ is the endpoint energy, θ is the angle between $\mathbf{p_e}$ and the spin orientation axis, and $\mathcal{A}$ is the nuclear alignment. The difference $\alpha_- - \alpha_+$ is sensitive to the weak magnetism term as well as to the induced tensor term in the nucleon weak current. Unfortunately, it is also sensitive to the difference of the axial charges ($\Delta y \equiv y_+ - y_-$) in the mirror transitions $^{12}\mathrm{B} \to ^{12}\mathrm{C}$ and $^{12}\mathrm{N} \to ^{12}\mathrm{C}$ — this potentiality has been included in only the most recent set of SCC tests [7,8]. Were $\Delta y = 0$ and the experimental weak magnetism contribution determined from the M1 electromagnetic transition strength from the analog state of $^{12}\mathrm{C}$ [9], as per the CVC hypothesis, Refs. [7] and [8] would yield $2Mf_T/f_A = 0.12 \pm 0.05(\mathrm{stat}) \pm 0.15(\mathrm{syst})$ and $2Mf_T/f_A = 0.04 \pm 0.16(\mathrm{stat}) \pm 0.04(\mathrm{syst})$, respectively. Note that f_T and f_A denote the induced-tensor and axial-vector coupling constants of the nucleon — the impulse approximation has been made in order to relate the nuclear and nucleon weak constants, note, e.g., Ref. [10]. This is consistent with the earlier result $2Mf_T/f_A = -0.21 \pm 0.63$ [10]. Using $\Delta y = 0.10 \pm 0.05$ [11], Refs. [7] and [8] determine that $2Mf_T/f_A = 0.22 \pm 0.05(\mathrm{stat}) \pm 0.15(\mathrm{syst}) \pm 0.05(\mathrm{theor})$ and $2Mf_T/f_A = 0.14 \pm 0.16(\mathrm{stat}) \pm 0.04(\mathrm{syst}) \pm 0.05(\mathrm{theor})$, yielding the combined constraint $0.01 \le 2Mf_T/f_A \le 0.34$ at 90% CL [8]. This result suggests that f_T is non-zero, with a value considerably in excess of SM expectations [12,13]. The inferred SCC contribution emerges from assuming the CVC hypothesis; alternatively, we can assert that SCC are identically zero in order to ascertain the quantitative validity of the CVC hypothesis. The uncertainties

*E-mail: gardner@pa.uky.edu

in the SCC determination are roughly 5% of the value of the weak magnetism contribution, so that the CVC hypothesis is tested to this level. Note that an analogous test of SCC is possible in the mass 8 system as well. Combining the radiative decays of the analog doublet in ^{8}Be [14,15] with measurements of the $\beta-\alpha$ correlation in ^{8}Li $\to^8$Be and ^{8}B $\to^8$Be decays [16] yields a second-class, induced tensor nuclear form factor which is consistent with zero [15], albeit with an error rather larger than in the mass 12 system. The mass 8 CVC/SCC studies ought also to suffer a theoretical correction from the difference in the allowed axial matrix elements in the mirror ^{8}Li $\to^8$Be and ^{8}B $\to^8$Be decays; the induced tensor form factor of Ref. [15] assumes this correction is zero.

We believe that a crisper test of the CVC hypothesis and of the non-existence of SCC can be realized via the empirical determination of the correlation coefficients of neutron β-decay. Thus far, the especial focus of these experiments has been the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ud}. The latter is extracted from g_V, which is determined from the neutron-spin–electron-momentum correlation A and the neutron lifetime τ_n. The various determinations of A do not agree [17]; a scale factor of 1.9 is assigned to the determination of g_A/g_V from the measured values of A by Ref. [18]. These measurements were realized in reactor beam experiments; A can also be measured using ultra-cold neutron sources — the systematic errors in such experiments are very different and would seem to be much smaller [19]. Nevertheless, the extracted value of V_{ud}, in concert with V_{us} from K_{l3} decays, tests the "squashed" unitarity relation $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=1$ to better than 1%. V_{ud} may also be determined, indeed, more precisely, from the "superallowed" $0^+\to 0^+$ decays in nuclei. In this case the empirical unitarity relation deviates from unity by 2.2σ; it is worth noting, however, that in this case the estimated theoretical errors dominate the presumed error bar [20].

Let us consider the correlation coefficients in neutron β-decay. The differential decay rate of a free neutron is given by [21]:

$$d\Gamma \propto E_e|\mathbf{p_e}|(E_e^{\max}-E_e)^2[1+a\tfrac{\mathbf{p_e}\cdot\mathbf{p}_\nu}{E_eE_\nu}+A\tfrac{\mathcal{P}\cdot\mathbf{p_e}}{E_e} + B\tfrac{\mathcal{P}\cdot\mathbf{p}_\nu}{E_\nu}+D\tfrac{\mathcal{P}\cdot(\mathbf{p_e}\times\mathbf{p}_\nu)}{E_eE_\nu}]dE_e d\Omega_e d\Omega_\nu, \quad (1)$$

where $\mathcal{P}$ denotes the neutron's polarization vector. The pseudo-T-odd coefficient D is small [22] and can be neglected. Defining $\lambda\equiv|g_A|/|g_V|$ and neglecting terms of recoil order we have in the SM

$$a=\frac{1-\lambda^2}{1+3\lambda^2} \quad ; \quad A=2\frac{\lambda(1-\lambda)}{1+3\lambda^2}\,, \quad B=2\frac{\lambda(1+\lambda)}{1+3\lambda^2}\,. \quad (2)$$

These relations imply that [23]

$$1+A-B-a=0\,, \quad (3)$$

$$aB-A-A^2=0. \quad (4)$$

Currently [18]

$$a=-0.102\pm0.005 \quad ; \quad A=-0.1162\pm0.0013, \quad B=0.983\pm0.004\,, \quad (5)$$

so that Eqs. (3) and (4) are satisfied at the current level of precision. However, these relations do not hold once terms of recoil order are included. The recoil-order terms are controlled by the dimensionless ratio of the electron energy to the neutron rest mass and thus are of $\mathcal{O}(10^{-3})$, so that they impact a and A at the 1% level. The correlation coefficient B is much larger, so that the recoil order terms only become important at the 0.1% level. Consequently we will focus on what can be learned from a and A. Recent experimental proposals suggest that A and possibly a can be measured to 0.2% or better [19,24]. We wish to point out that additional Standard Model tests are possible once terms of recoil order become empirically accessible. In particular, one is sensitive to both the weak magnetism term f_2 as well as to the induced tensor term g_2 in the nucleon weak current. Indeed, independent tests of the CVC hypothesis and of the non-existence of SCC are possible, as we shall now see.

The matrix element for polarized neutron β-decay in the SM is given by

$$\mathcal{M}=\frac{G_F}{\sqrt{2}}\langle p|J^\mu(0)|\vec{n}\rangle\times[\bar{u}_e(p_e)\gamma_\mu(1+\gamma_5)u_\nu(p_\nu)]\,. \quad (6)$$

We adopt the historic $(1+\gamma_5)$ sign convention in order to retain manifest consistency with earlier work [25–27]. Lorentz invariance and translation invariance implies that the nucleon weak current $\langle p|J^\mu(0)|\vec{n}\rangle$ has six terms:

$$\langle p(p')|J^\mu(0)|\vec{n}(p,\mathbf{s})\rangle=\bar{u}_p(p')[f_1(q^2)\gamma^\mu-i\frac{f_2(q^2)}{M}\sigma^{\mu\nu}q_\nu + \frac{f_3(q^2)}{M}q^\mu+g_1(q^2)\gamma^\mu\gamma_5-i\frac{g_2(q^2)}{M}\sigma^{\mu\nu}\gamma_5 q_\nu + \frac{g_3(q^2)}{M}\gamma_5 q^\mu]u_n(p,\mathbf{s}), \quad (7)$$

where $\sigma^{\mu\nu}=\frac{i}{2}[\gamma^\mu,\gamma^\nu]$ and $q=p-p'$. Note that $f_1(0)=g_V$, $g_1(0)=-g_A=-f_A/G_F$, and $g_2(0)=-f_T\,M/G_F$, whereas M and M' are the neutron and proton mass, respectively. The differential decay rate is given by

$$d^3\Gamma=\tfrac{|G_F|^2}{2(2\pi)^5}\tfrac{|\mathbf{p_e}||\mathbf{p}_\nu|}{M-E_e+|\mathbf{p_e}|\cos\theta}[C_1+C_2(\mathcal{P}\cdot\mathbf{p}_e) + C_3(\mathcal{P}\cdot\mathbf{p}_\nu)+C_4\mathcal{P}\cdot(\mathbf{p_e}\times\mathbf{p}_\nu)]dE_e d\Omega_e d\Omega_\nu\,, \quad (8)$$

where the coefficients C_i contain the form factors of Eq. (7) and are detailed in Ref. [25]. Note that θ is the angle between the electron and neutrino momenta in the neutron rest frame. Our particular interest are the recoil

corrections to a and A. Let us first consider the case in which the neutron is unpolarized. We have

$$d^2\Gamma = \tfrac{2|G_F|^2|g_V|^2}{(2\pi)^4}\tfrac{(MR)^4\beta x^2(1-x)^2}{(1-Rx+Rx\beta\cos\theta)^3}[C_a + C_b\beta\cos\theta]dE_e d\Omega_{e\nu}\ , \tag{9}$$

where

$$R = \frac{E_e^{\max}}{M} = \frac{1}{2}(1+\epsilon-\eta^2) \quad ; \quad x = \tfrac{E_e}{E_e^{\max}}\ , \tag{10}$$
$$\eta = \frac{M'}{M} \quad ; \quad \epsilon = (\tfrac{m_e}{M})^2$$

and $C_a + C_b\cos\theta = C_1/(2ME_\nu E_l|g_V|^2)$ — C_1 contains the electron-anti-neutrino correlation, a. Working in leading recoil order, including the phase space contributions, we have

$$d^2\Gamma = \tfrac{2|G_F|^2|g_V|^2}{(2\pi)^4}(MR)^4\beta x^2(1-x)^2[\tilde{C}_a + \tilde{C}_b\beta\cos\theta + \tilde{C}_c\beta^2\cos^2\theta]dE_e d\Omega_{e\nu}\ , \tag{11}$$

where

$$\begin{aligned}\tilde{C}_a &= 1+3\lambda^2 - \frac{\epsilon}{Rx}(1+2\lambda+\lambda^2+4\tilde{f}_2\lambda+2\tilde{g}_2\lambda\\ &\quad -2\tilde{f}_3) - R(2\lambda^2+2\lambda+4\tilde{f}_2\lambda+4\lambda\tilde{g}_2)\\ &\quad +Rx(3+9\lambda^2+4\lambda+8\tilde{f}_2\lambda)\ , \qquad (12)\\ \tilde{C}_b &= 1-\lambda^2+R(2\lambda+2\lambda^2+4\tilde{f}_2\lambda+4\lambda\tilde{g}_2)\\ &\quad -4Rx(\lambda+3\lambda^2+2\tilde{f}_2\lambda)\\ \tilde{C}_c &= -3Rx(1-\lambda^2)\end{aligned}$$

with $\tilde{f}_2 \equiv f_2(0)/f_1(0)$ and $\tilde{g}_2 \equiv g_2(0)/f_1(0)$. The momentum dependence of the form factors does not appear, as this effect first enters in next-to-leading recoil order. Noting Eq. (1) we have $a = \tilde{C}_b/(\tilde{C}_a + \tilde{C}_c\beta^2\cos^2\theta)$ and thus

$$\begin{aligned}a=& \frac{1-\lambda^2}{1+3\lambda^2} + \frac{1}{(1+3\lambda^2)^2}\Bigg\{\frac{\epsilon}{Rx}\Big[(1-\lambda^2)(1+2\lambda+\lambda^2\\ &+2\lambda\tilde{g}_2+4\lambda\tilde{f}_2-2\tilde{f}_3)\Big] + 4R\Big[(1+\lambda^2)(\lambda^2+\lambda\\ &+2\lambda(\tilde{f}_2+\tilde{g}_2))\Big] - Rx\Big[3(1+3\lambda^2)^2+8\lambda(1+\lambda^2)\\ &\times(1+2\tilde{f}_2)+3(\lambda^2-1)^2\beta^2\cos^2\theta\Big]\Bigg\} + \mathcal{O}(R^2,\epsilon)\ . \qquad (13)\end{aligned}$$

If $\tilde{f}_3 = \tilde{g}_2 = 0$, this expression becomes that of Ref. [26]. Note, too, that it is also in agreement with Ref. [28].

The recoil correction to A is determined from Eq.(8) by integrating over the neutrino variables. We find [25]

$$d^2\Gamma = \frac{2|G_F|^2|g_V|^2}{(2\pi)^3}\frac{\beta x^2(1-x)^2}{(1+\epsilon-2Rx)^3}[C'_a + C'_b\beta\mathcal{P}\cos\theta_{\mathcal{P}}]dE_e d(\cos\theta_{\mathcal{P}})\ , \tag{14}$$

where $\theta_{\mathcal{P}}$ is the angle between the neutron's polarization vector and the electron momentum in the neutron rest frame. C_2 and C_3 give rise to C'_b, whereas C_1 gives rise to C'_a. Noting $A = C'_b/C'_a$, we have

$$\begin{aligned}A=& \frac{2\lambda(1-\lambda)}{1+3\lambda^2} + \frac{1}{(1+3\lambda^2)^2}\Bigg\{\frac{\epsilon}{Rx}\Big[4\lambda^2(1-\lambda)(1+\lambda\\ &+2\tilde{f}_2)+4\lambda(1-\lambda)(\lambda\tilde{g}_2-\tilde{f}_3)\Big] + R\Big[\frac{2}{3}(1+\lambda\\ &+2(\tilde{f}_2+\tilde{g}_2))(3\lambda^2+2\lambda-1)\Big] + Rx\Big[\frac{2}{3}(1+\lambda+2\tilde{f}_2)\\ &\times(1-5\lambda-9\lambda^2-3\lambda^3)+\frac{4}{3}\tilde{g}_2(1+\lambda+3\lambda^2+3\lambda^3)\Big]\Bigg\}\\ &+\mathcal{O}(R^2,\epsilon)\ . \qquad (15)\end{aligned}$$

If $\tilde{f}_3 = \tilde{g}_2 = 0$, this expression becomes that of Refs. [26,29]. Our result is also in agreement with Ref. [28]. Our results are germane to hyperon decay as well; in this context either approximate expressions or the E_e-integrated asymmetry parameters are reported [30]. The recoil corrections to the correlation coefficients take the form: $a_0R+a_1Rx+a_{-1}\epsilon/Rx$. The energy dependence of the three terms is distinct, although only two terms are empirically accessible as $\epsilon/R \sim 2.2\cdot10^{-4}$, whereas $R \sim 1.4\cdot10^{-3}$. Note that $x\in[\sqrt{\epsilon}/R, 1]$. Thus we have four independent empirical constraints, i.e., the x^0 and x^1 terms in a and A, and three unknowns — namely, λ, $\tilde{f}_2$, and $\tilde{g}_2$. The system is overconstrained, so that we can infer the existence of physics beyond the SM, namely the presence of non-$V-A$ currents [21], if the extracted coupling constants differ from SM bounds or if the values of the extracted couplings are not consistent with each other. Note that independent linear combinations of $\tilde{f}_2$ and $\tilde{g}_2$ appear in a and A, so that, unlike the nuclear cases commonly studied, each coupling constant can be determined independently. Evaluating the recoil-order contributions to a and A, using $\lambda = 1.2670$ [18], $\tilde{g}_2 = 0$, and $\tilde{f}_2 = (\kappa_p - \kappa_n)/2 = 1.8529$, as per the CVC hypothesis, we find that the recoil corrections to a are roughly a factor of two larger than those to A. By virtue of the allowed terms, λ is determined to 0.030% and 0.022% by 0.1% measurements of a and A, respectively. On statistical grounds, a precision measurement of A would be the most efficacious in determining λ, whereas the determination of the coupling constants appearing in recoil order would seem to be better served with an a measurement. $\tilde{f}_2$ and $\tilde{g}_2$ can be determined in a plurality of ways; let us illustrate. Firstly, the x^1 and x^0 terms in a can be determined to yield $\tilde{f}_2$ and $\tilde{f}_2+\tilde{g}_2$. λ will be sufficiently precisely determined to have little impact on the errors in these parameters. Ignoring this source of error, we find a 0.1% measurement of a yields a 2.5% measurement of $\tilde{f}_2$ from the x^1 term. This, in concert with the x^0 term from a 0.1% measurement of a, yields an uncertainty in $\tilde{g}_2$ of order $0.22\lambda/2$ — this is compatible with the errors

quoted in the mass 12 experiment with far fewer assumptions. Secondly, the x^1 dependence of the a and A terms can be determined – the former yields $\tilde{f}_2$, whereas the latter yields a combination of $\tilde{f}_2$ and $\tilde{g}_2$ [31]. Earlier determinations of a were inferred from the recoil proton's spectral shape, see, e.g., Ref. [32], and were insensitive to the x dependence of a; the newly proposed a experiment [24] would be the first to measure a as a function of x [33]. The Fierz interference term, b [21], which is zero in the SM can thus be bounded as well. Combining the earlier determination of $\tilde{f}_2$ with a 0.1% measurement of A to determine $\tilde{g}_2$ from the x^1 term yields an uncertainty of $0.26\lambda/2$, commensurate with our earlier estimate. Although 0.1% measurements of A seem quite feasible [34], measurements of a to better than 1% may pose an especial challenge. Nevertheless, precision measurements of a and A are richly complementary. The measurement of both a and A permit crisp SM tests, namely of SCC and the CVC hypothesis, not realizable in nuclear decays.

We thank M. S. Dewey and F. Wietfeldt for introducing us to the NIST "a" experiment, and we gratefully acknowledge F. Wietfeldt, E. Adelberger, B. Filippone, B. Holstein, R. McKeown, K. Snover, and J. Tandean for additional helpful conversations and correspondence. We acknowledge the support of the DOE under contract number DE-FG02-96ER40989.

[1] M. Gell-Mann, Phys. Rev. **111**, 362 (1958).
[2] S. Weinberg, Phys. Rev. **112**, 1375 (1958).
[3] E. D. Commins and P. H. Bucksbaum, *Weak Interactions of Leptons and Quarks*, Cambridge University Press, Cambridge, 1983.
[4] L. Grenacs, Ann. Rev. Nucl. Part. Sci. **35**, 455 (1985).
[5] L. De Braeckeleer, Phys. Rev. C **45**, 1935 (1992).
[6] If we denote p_m to be the fractional population in the $m = 1, 0, -1$ magnetic substates, then the polarization $\mathcal{P} \equiv p_1 - p_{-1}$ and the alignment $\mathcal{A} \equiv p_1 + p_{-1} - 2p_0$. A "purely aligned" target has $\mathcal{P} = 0$.
[7] T. Minamisono *et al.*, Phys. Rev. Lett. **80**, 4132 (1998).
[8] K. Minamisono *et al.*, Nucl. Phys. **A663**, 951 (2000).
[9] K. Koshigiri *et al.*, Nucl. Phys. **A319**, 301 (1979); J. Phys. Soc. Jpn. Suppl. **55**, 1014 (1986).
[10] Y. Masuda *et al.*, Phys. Rev. Lett. **43**, 1083 (1979) and references therein.
[11] K. Koshigiri *et al.*, Nucl. Phys. **A588**, 165c (1995).
[12] J. F. Donoghue and B. R. Holstein, Phys. Rev. **D25**, 206 (1982).
[13] H. Shiomi, Nucl. Phys. **A603**, 281 (1996).
[14] L. De Braeckeleer et al., Phys. Rev. **C51**, 2778 (1995).
[15] K. Snover *et al.*, Internat. Symp. of Nuclear Electroweak Spectroscopy (NEWS99), Osaka, Japan, March 1999, World Scientific, to be published.
[16] R. E. Tribble and G. T. Garvey, Phys. Rev. **C12**, 967 (1975); R. D. McKeown, G. T. Garvey, and C. A. Gagliardi, Phys. Rev. **C22**, 738 (1980); M. Beck *et al.*, in *Intersections Between Particle and Nuclear Physics,* 6th *Conference*, Big Sky, MT, May 27 - June 2, 1997, AIP Conf. Proc. 412, T. W. Donnelly, ed. (AIP Press, New York, 1997), p. 416.
[17] H. Abele et al., Phys. Lett. **B407**, 212 (1997); P. Liaud et al., Nucl. Phys. **A612**, 53 (1997); B.G. Erozolimsky et al., Phys. Lett. **B412**, 240 (1997); P. Bopp et al., Phys. Rev. Lett. **56**, 919 (1986).
[18] D. E. Groom *et al.* [Particle Data Group], Eur. Phys. J. **C15**, 1 (2000).
[19] A Proposal for an Accurate Measurement of the Neutron Spin-Electron Angular Correlation in Polarized Neutron Beta Decay with Ultra-Cold Neutrons, T. Bowles and A.R. Young, co-Principal Investigators.
[20] I. S. Towner and J. C. Hardy, nucl-th/9809087.
[21] J.D. Jackson, S.B. Treiman, and H. W. Wyld, Jr., Phys. Rev. **106**, 517 (1957); Nucl. Phys. **4**, 206 (1957).
[22] L. J. Lising *et al.* [emiT Collaboration], Phys. Rev. **C62**, 055501 (2000).
[23] Yu. A. Mostovoĭ and A. I. Frank, JETP Lett. **24**, 38 (1976).
[24] F. Wietfeldt et al., Fundamental Physics with Pulsed Neutron Sources Conference, Durham, NC (July 1-2, 2000), to be published.
[25] D. R. Harrington, Phys. Rev. **120**, 1482 (1960). Minor errors in the C_i impact only the f_3 contribution to A and a in leading recoil order. The expressions reported for C'_a and C'_b and consequently for the recoil correction to A are inconsistent with ours.
[26] S. M. Bilen'kiĭ et al., Sov. Phys. JETP **37**, 1241 (1960).
[27] I. Bender, V. Linke, and H. J. Rothe, Z. Phys. **212**, 190 (1968).
[28] B. R. Holstein, Rev. Mod. Phys. **46**, 789 (1974); Erratum, **48**, 673 (1976). Effecting the correspondence $a = f_1$, $b = \sqrt{3}(f_1 + 2f_2)$, $c = \sqrt{3}g_1$, $d = 2\sqrt{3}g_2$, $e = 2f_3$, and $h = 2\sqrt{3}g_3$, we find that the $f_i(E)$ of Eq.(51) are given by $f_1(E) + (f_1^2 - g_1^2)(Rx - \epsilon/(Rx)) = \tilde{C}_a$, $f_2(E) = \tilde{C}_b$, and $f_3(E) = \tilde{C}_c$. The corrected F_i of Eq.(52) are consistent with our C'_a and C'_b once the recoil-order terms from the phase space factor $(1 + \epsilon - 2Rx)^{-3}$ are incorporated.
[29] D. H. Wilkinson, Nucl. Phys. **A377**, 474 (1982).
[30] V. Linke, Nucl. Phys. **B12**, 669 (1969); J. M. Watson and R. Winston, Phys. Rev. **181**, 1907 (1969). Comparing Eq. (11) with the approximate treatment of the latter we find that the terms involving $\tilde{f}_2$ and $\tilde{g}_2$ in R and Rx are identical, whereas the others differ. See also P. H. Frampton and W. K. Tung, Phys. Rev. **D3**, 1114 (1971); Y. Yokoo, S. Suzuki, and M. Morita, Prog. Theor. Phys. **50**, 1894 (1973). Note that Ref. [27] treats the unpolarized case; our C'_a can be recovered from their expressions.
[31] Ref. [28] notes, at the bottom of p. 805, that the b and d terms can be determined independently from the energy dependence of the electron-antineutrino correlation and polarization asymmetry in an analog decay, such as $^{19}\mathrm{Ne} \rightarrow ^{19}\mathrm{F}$ or $n \rightarrow p$.
[32] C. Stratowa, R. Dobrozemsky, and P. Weinzierl, Phys. Rev. **D18**, 3970 (1978).
[33] F. Wietfeldt, private communication.
[34] B. Filippone, private communication.